**Propositional Stability[[1]](#footnote-1)**

Adam InTae Gerard[[2]](#footnote-2)

1. **Introduction**

This short article adumbrates a new and useful notion relevant to so-called combined modal logics, Markov Logic Networks, and Transactional Logic (forthcoming). Specifically, we seek to define and identify the conditions under which truth-values remain stable when interacted with by **more than one logic**.

“Under what conditions”, we might ask, “do propositions remain unchanged in their truth-determinations?” Furthermore, “how might we proceed to calculate that and track such changes?” Propositional Stability is introduced to that end.

**1.0 Overview and Motivation**

Post-truth, subjectivism, post-modernity, anti-rationalism, anti-intellectualism, memetics, black-boxed artificial intelligence, iterative logics (logics that fail to exhibit eternalism), hyper-dimensional logics (forth-coming), logical pluralism, substructural logics, logics of contradiction and paradox, declassified UFO’s, and constructive mathematics.

Formally, Propositional Stability ensures that when a proposition is transacted between two logics (more on this later) - it never acquires a new truth-value beyond those it could have already acquired under the first logic under which it is evaluated.

**2.0 Conventions**

Where:

1. ⊶ ∈ ℕ
2. ⋇ ∈ {*a*, ..., *z*, ...}
3. {*a*, ..., *z*, ...} = ℕ

We write (quotes[[3]](#footnote-3) are dropped):

1. *ML*⊶⋇ to denote a semantics (model or truth-assignment *M*) for a language *L*⊶ with ⋇-many truth values.
2. *VML*⊶⋇(p) to denote a truth-evaluation of *p* under semantics (model or truth-assignment *M*) for a language *L*⊶ with ⋇-many truth values.
3. *VML1aVML2b*(p)\* to denote any possible truth-evaluation of p to a truth-value t in semantics *ML2b* such that: *t* ∈ *ML2b* and *t* ∉ *ML1a*.

**3.0 Definitions**  
  
**Definition 1.** *Instruction set*.

An **instruction set** is a finite procedure or algorithm mapping one input to one output.   
  
**Definition 2.** *Strong propositional stability*.

1. A proposition or sentence p evaluated under semantics *ML1a* will preserve its exact truth-value under semantics *ML2b* whenever *a* ⊆ *b* and no instruction set exists to map *VML1a*(p) to any other truth-value. p is then said to exhibit ***strong*** ***propositional stability***.
2. A proposition *p* exhibits ***strong*** ***propositional stability*** *when and only when*:
   1. *VML1a*(p) = *VML2b(p)*
   2. *t* ∈ *VML1a* ∪ *VML2b*
   3. *VML1a*(p) ≠ *t*
   4. No instruction set exists to map *VML1a*(p) to *t*

**Definition 3.** *Weak propositional stability*.

1. A proposition or sentence p evaluated under semantics *ML1a* will preserve its range of truth-values under semantics *ML2b* whenever *a* ⊆ *b* and no instruction set exists to map *VML1a*(p) to any *VML1aVML2b*(p)\*. p is then said to exhibit ***weak*** ***propositional stability***.
2. A proposition *p* exhibits ***weak*** ***propositional stability*** *when and only when*:
   1. *VML1a*(p) ⊆ *VML2b(p)*
   2. No instruction set exists to map *VML1a*(p) to any *VML1aVML2b*(p)\*.

**Definition 4**. Truth stability.

1. A proposition or sentence p evaluated under semantics *ML1a* will preserve its exact truth-value under semantics *ML2b* whenever *a* ⊆ *b*. p is then said to exhibit ***truth stability***.
2. A proposition *p* exhibits ***truth stability*** *when and only when* *VML1a*(p) = *VML2b(p)*.

**Definition 5**. Propositional instability.

A proposition *p* exhibits ***propositional instability*** whenever it does not exhibit ***weak propositional stability***.

**Definition 6**. Truth instability.

A proposition *p* exhibits ***truth instability*** whenever it does not exhibit ***truth stability***.

**4.0 Discussion**

**Remark 1. *Strong propositional stability*** entails ***weak propositional stability*** and **truth stability**.

**Discussion:** ***Strong propositional stability*** requires that a proposition retains its exact truth-value under two logics and that no method exists for that truth-value to vary. Thus, it is constrained by the same range of truth-values.

**Remark 2. Truth stability** guarantees only incidental sameness of truth-assignment. In some cases, truth stability will converge with ***strong propositional stability***, in others it will not.

**5.0 Results**

**Fact 1.** Any proposition truth-evaluated under a Boolean logic will exhibit ***strong* propositional stability** when truth-evaluated under a Kleene 3-Value Algebra.

**Proof:** Obvious. No single proposition already assigned a truth-value of 'true' or 'false' can receive a truth-value of 'indeterminate' or 'true and false'. ∎

**Fact 2.** Given:

1. Monotonic axiom systems Ω1, Ω2
2. Ω1 ⊂ Ω2

If Ω1 ├ *A*, *A* will exhibit ***strong propositional stability*** under Ω2.

**Proof:** Obvious. If Ω1 ├ *A*, then Ω2 ├ *A*. *A* will remain a derived tautology under Ω2. ∎

**Fact 3.** Given:

1. Monotonic axiom systems Ω1, Ω2
2. Ω1 ⊂ Ω2
3. Γ├ *A*

If Ω2 ├ *A* and Γ ⊆ Ω2, *A* will exhibit:

1. ***Strong propositional stability*** under Ω1 only when Γ ⊆ Ω1
2. ***Propositionally instability*** otherwise.

**6.0 Modal Logic and Axioms Systems**

… TBD about combine modal logics.

**7.0 Conclusion**

Here and elsewhere, I have asserted that the fundamental concepts currently in wide-spread use throughout mathematics, philosophy, science, finance, ethics, law, and so on all largely rely on *ontological dogmas* including truth-monism, classicality, the T-Schema, and objecthood. While I will not argue on the subject here, *propositional stability* remains of interest to all such considerations.

**A.0 Appendix**

Originally Posted at: <http://www.postlib.com/propositional-stability/>

1. Work in Progress – Under Heavy Revision [↑](#footnote-ref-1)
2. **Revision 0.0.3** – **10.13.18** - <https://www.linkedin.com/in/adamintaegerard/> [↑](#footnote-ref-2)
3. <https://plato.stanford.edu/entries/quotation/#2.2> [↑](#footnote-ref-3)